

Confidence Intervals for Means

Lecture 36 Section 10.4

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Outline

Confidence Intervals for Means

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How to Choose the Right Statistic

Examples

Using Z

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- We close out Chapter 10 by computing confidence intervals for μ .
- The basic theory is the same as it was for p .
- We will face the same choice between Z and t that we faced in hypothesis testing for μ .
- We will see that the TI-83 will handle the calculations for us, provided we make the right choices.

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- To estimate μ , we will use confidence intervals, as we did when estimating p .
- With p , we used \hat{p} as a point estimate, computed $\sigma_{\hat{p}}$, and found

$$\hat{p} \pm z\sigma_{\hat{p}}.$$

- In general, to find a confidence interval, we begin with a point estimate e of the parameter of interest, find the standard deviation σ_e of that estimator, and compute

$$e \pm (\text{appropriate coeff.})\sigma_e.$$

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Summary

- What is the point estimate for μ ?
- What is the standard deviation for this estimator?
- How do we determine the appropriate number of standard deviations?
- We need to know the appropriate underlying distribution of the estimator.
- The distribution will be either z or t , depending on the circumstances.

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Summary

- The confidence interval will be

$$\bar{x} \pm z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right),$$

or

$$\bar{x} \pm z_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right),$$

or

$$\bar{x} \pm t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right),$$

depending on the circumstances.

When to Use Z

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- Use Z if
 - The population is normal (or nearly normal) and σ is known, **or**
 - The sample size is at least 30.

When to Use t

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Summary

- Use t if
 - The population is normal (or nearly normal), **and**
 - σ is not known.

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- Example 10.4, p. 641.
- The weights of a certain kind of breakfast cereal have a normal distribution with $\sigma = 0.29$.
- A sample of 25 boxes of cereal has a mean weight of 9.82 oz.
- Construct a 95% confidence interval for the true mean weight of such boxes.

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- Should we use z or t ? Why?
- Should we use s or σ ? Why?
- $n = 25$.
- $\bar{x} = 9.82$.
- Level of confidence is 95%.

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- We should use z (even though $n < 30$) because σ is known and the population is normal.
- For 95% confidence, the value of z is 1.960.
- The confidence interval is

$$\bar{x} \pm z_{\alpha/2} \left(\frac{\sigma}{\sqrt{n}} \right) = 9.82 \pm 1.96 \left(\frac{0.29}{\sqrt{25}} \right) = 9.82 \pm 0.114.$$

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- Example 10.5, p. 643.
- A sample of 61 airline flights was taken.
- Average number of unoccupied seats was 12.6 with a standard deviation of 4.4 seats.
- Construct a 99% confidence interval for the mean number of unoccupied seats.

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- Should we use Z or t ? Why?
- Should we use s or σ ? Why?
- $n = 61$.
- $\bar{x} = 12.6$.
- $s = 4.4$.
- Level of confidence is 99%.

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- We should use t .
- What value of t cuts off a tail of area 0.005?
- That is, if $df = 60$ and $\alpha/2 = 0.005$, what is $t_{60,0.005}$?
- Unless you have the function `invT` on the TI-83, then you will have to use the t table.
- We look up $t_{60,0.005}$ (row 60, column 0.005) and find that t is 2.660.

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- The confidence interval is

$$\bar{x} \pm t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right) = 12.6 \pm 2.660 \left(\frac{4.4}{\sqrt{61}} \right) = 12.6 \pm 1.5.$$

TI-83 - Confidence Intervals

Using `ZInterval`

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Summary

- The TI-83 will compute confidence intervals for μ .
- When the standard normal distribution applies, do the following.
 - Press `STAT`.
 - Select `TESTS`.
 - Select `ZInterval`. A window appears requesting information.
 - Select `Data` or `Stats`.

TI-83 - Confidence Intervals

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- Assume we selected `Stats`.
 - Enter σ .
 - Enter \bar{x} .
 - Enter n .
 - Enter the level of confidence.
 - Select `Calculate` and press `ENTER`.

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- A window appears containing
 - The title `ZInterval`.
 - The confidence interval in interval notation.
 - The sample mean.
 - The sample size.

TI-83 - Confidence Intervals

Using `ZInterval`

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- Re-do the earlier example on the TI-83.

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- When the t distribution applies, do the following.
 - Press STAT.
 - Select TESTS.
 - Select TInterval. A window appears requesting information.
 - Select Data or Stats.

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- Assume we selected `Stats`.
 - Enter \bar{x} .
 - Enter s .
 - Enter n .
 - Enter the level of confidence.
 - Select `Calculate` and press `ENTER`.

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- A window appears containing
 - The title TInterval.
 - The confidence interval in interval notation.
 - The sample mean.
 - The sample standard deviation.
 - The sample size.

TI-83 - Confidence Intervals

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- Re-do the earlier example on the TI-83.

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- Confidence intervals for μ are very similar to confidence intervals for p .
- The basic formula is

$$\bar{x} \pm t_{\alpha/2} \left(\frac{s}{\sqrt{n}} \right),$$

where t could be replaced by z and s could be replaced by σ , depending on the circumstances.

- Use the t formula if
 - The population is normal, and
 - σ is unknown.
- The TI-83 functions `ZInterval` and `TInterval` will compute confidence intervals for the mean using Z or t .